Deep Learning In An Afternoon For Physicists

John Urbanic Parallel Computing Scientist Pittsburgh Supercomputing Center

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Who am I?

John Urbanic Parallel Computing Scientist Pittsburgh Supercomputing Center & CMU Physics

I am here to work with you on anything high performance computing related. Which includes many things - like this.

7311 Wean Hall and via <u>urbanic@psc.edu</u>

any time. And I look forward to resuming office hours.

NSF Monthly Workshop Series

- September 3-4 HPC Monthly Workshop: MPI
- October 1-2 HPC Monthly Workshop: Big Data
- November 5 HPC Monthly Workshop: OpenMP
- December 3-4 HPC Monthly Workshop: Big Data
- January 21 HPC Monthly Workshop: OpenMP
- February 19-20 HPC Monthly Workshop: Big Data
- March 3 HPC Monthly Workshop: OpenACC
- April 7-8 HPC Monthly Workshop: Big Data
- May 5-6 HPC Monthly Workshop: MPI
- June 2-5 Summer Boot Camp
- August 10-11 HPC Monthly Workshop: Big Data
- September 14-15 HPC Monthly Workshop: MPI
- October 5-6 HPC Monthly Workshop: Big Data
- November 3 HPC Monthly Workshop: OpenMP
- December 7-8 HPC Monthly Workshop: Big Data



Logistics

Schedule1:00Start3:0020 Minute Break5:00Finish

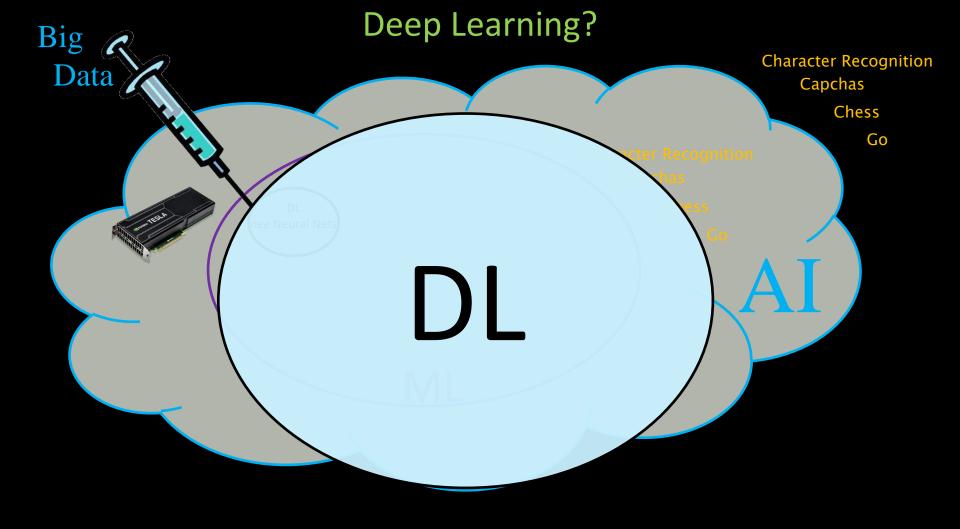
Materials http://psc.edu/dl-for-physicists

Questions

- I hope you have lots.
- Let's start with chat and hands-up for mic.
- We have a producer to help manage.

Hands-On When we get there, but mostly after.





For Physicists?

Is this all about physics applications?

IOP Conf. Series: Journal of Physics: Conf. Series 1085 (2018) 042022 doi:10.1088/1742-6596/1085/4/042022

End-to-End Event Classification of High-Energy Physics Data

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Abstract. Feature extraction algorithms, such as convolutional neural networks, have introduced the possibility of using deep learning to train directly on raw data without the need for rule-based feature engineering. In the context of particle physics,

Category

CNN

FCN

7x7 conv, 16, /2

ResBlock x 5, 16

ResBlock x 1, 32, /2 ¥ ResBlock x 5, 32

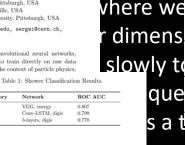
(a) ResNet-23

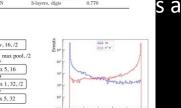
LSTM

such end-to-end approaches can from detector-level data in a way physics reconstruction. We demons classifiers to distinguish simulated the CMS Electromagnetic Calorim

1. Introduction & Motivation

An essential part of any new physics searce CERN involves event classification, or distin Traditional machine learning techniques

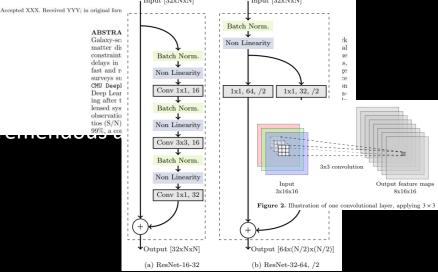




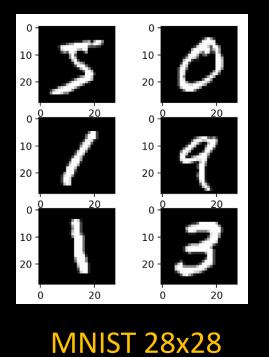
(b) e^+e^- vs. $\gamma\gamma$ Classifier Output

Classifier Output





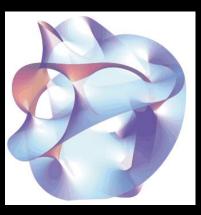
Why Would An Image Have 784 Dimensions?



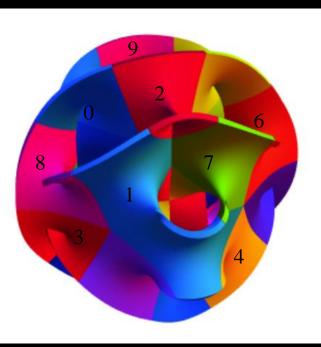
greyscale images

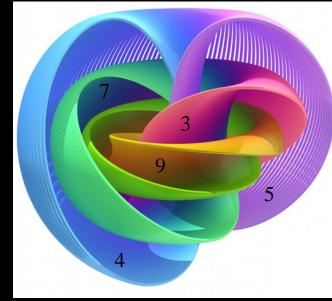
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										169	253	253	253	253	253	253	218											o
5 -								0	169	253	253	253	213	142	176	253	253	122										o
								52	250	253	210	32	12	0	6	206	253	140										o
							o	77	251	210	25			0	122	248	253	65										o
								0	31	18				0	209	253	253	65										o
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											0	176	246	253	159													0
											25	234	253	233														o
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15 -										78	248	253	189															0
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20 -	0						0	0	174	253	253	253	253	253	253	253	253	253	253	253	253	253	253	169		117	57	0
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Central Hypothesis of Modern DL



Data Lives On A Lower Dimensional Manifold





Maybe Less So

Maybe Very Contiguous

Dimensionality Reduction

You will find a recurring theme throughout machine learning, not just deep learning:

- Our data naturally resides in higher dimensions
- Reducing the dimensionality makes the problem more tractable
- And simultaneously provides us with insight

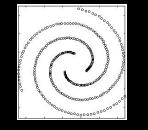
This last two bullets highlight the principle that "learning" is often finding an effective compressed representation.

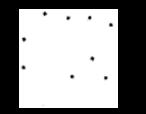
Clustering

As intuitive as clustering is, it presents challenges to implement in an efficient and robust manner.

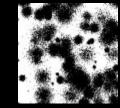
You might think this is trivial to implement in lower dimensional spaces.

But it can get tricky even there.





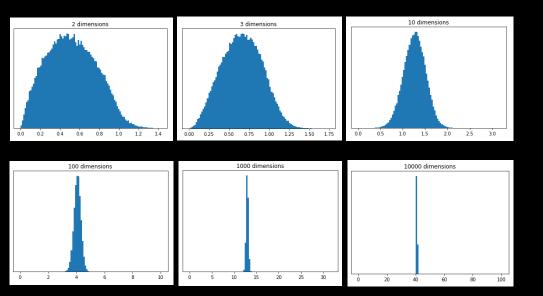
Sometimes you know how many clusters you have to start with. Often you don't. How hard can it be to count clusters? How many are here?



And in much higher dimensional spaces we run into the Curse of Dimensionality.

Curse of Dimensionality

This is a good time to point out how our intuition can lead us astray as we increase the dimensionality of our problems - which we will certainly be doing - and to a great degree. There are several related aspects to this phenomenon, often referred to as the *Curse of Dimensionality*. One root cause of confusion is that our notion of Euclidian distance starts to fail in higher dimensions.



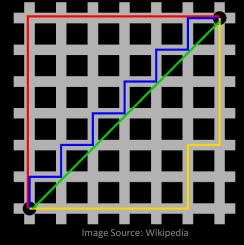
These plots show the distributions of pairwise distances between randomly distributed points within differently dimensioned unit hypercubes. Notice how all the points start to be about the same distance apart.

Once can imagine this makes life harder on a clustering algorithm!

There are other surprising effects: random vectors are almost all orthogonal; the unit sphere takes almost no volume in the unit square. These cause all kinds of problems when generalizing algorithms from our lowly 3D world.

Metrics

Even the definition of distance (the *metric*) can vary based upon application. If you are solving chess problems, you might find the Manhattan distance (or taxicab metric) to be most useful.

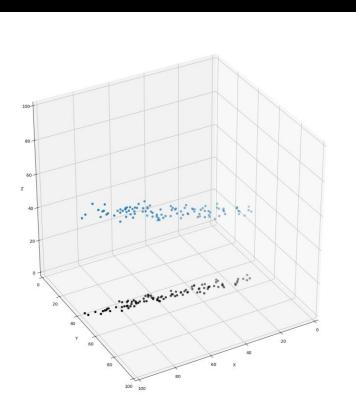


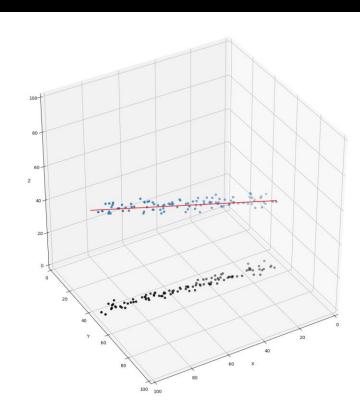
For comparing text strings, we might choose one of dozens of different metrics. For spell checking you might want one that is good for phonetic distance, or maybe edit distance. For natural language processing (NLP), you probably care more about tokens.

For genomics, you might care more about string sequences.

Some useful measures don't even qualify as metrics (usually because they fail the triangle inequality: $a + b \ge c$).

Alternative DR: Principal Component Analysis

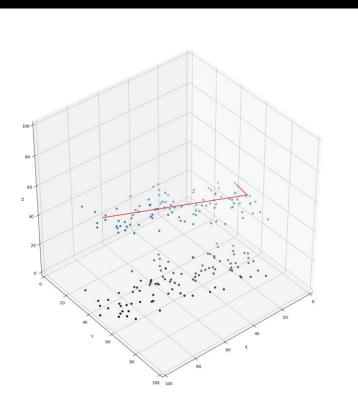


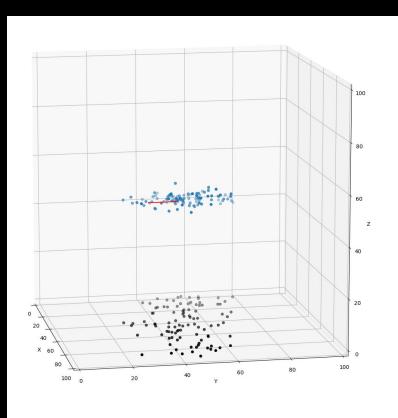


3D Data Set

Maybe mostly 1D!

Alternative DR: Principal Component Analysis





Flatter 2D-ish Data Set

View down the 1st Princ. Comp.

Testing These Ideas With Scikit-learn

import numpy as np import matplotlib.pyplot as plt from sklearn import (datasets, decomposition, manifold, random_projection) def draw(X, title): plt.figure() plt.xlim(X.min(0)[0],X.max(0)[0]); plt.ylim(X.min(0)[1],X.max(0)[1]) plt.xticks([]); plt.yticks([]) plt.title(title) for i in range(X.shape[0]): plt.text(X[i, 0], X[i, 1], str(y[i]), color=plt.cm.Set1(y[i] / 10.)) divide detasets land divide alere ()

digits = datasets.load_digits(n_class=6) X = digits.data y = digits.target

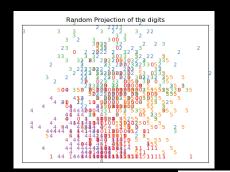
rp = random_projection.SparseRandomProjection(n_components=2, random_state=42)
X_projected = rp.fit_transform(X)
draw(X_projected, "Random Projection of the digits")

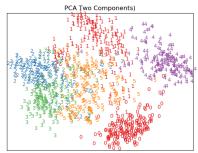
```
X_pca = decomposition.PCA(n_components=2).fit_transform(X)
draw(X_pca, "PCA (Two Components)")
```

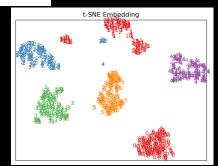
tsne = manifold.TSNE(n_components=2, init='pca', random_state=0)
X_tsne = tsne.fit_transform(X)
draw(X_tsne, "t-SNE Embedding")

plt.show()

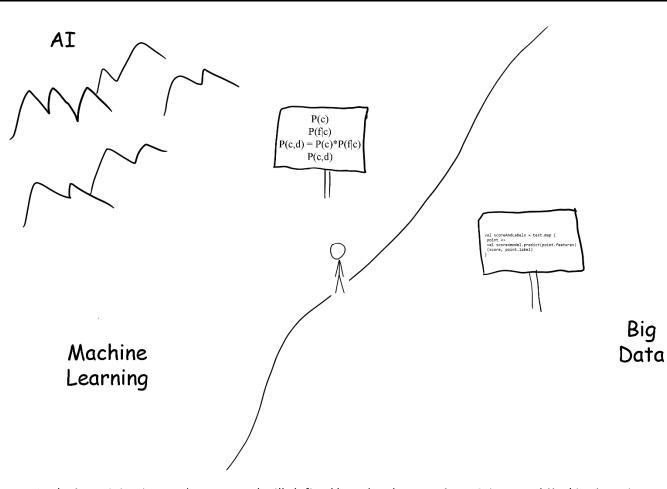
Sample of 64-dimensional digits dataset
01234501234501234505 55041351002220123333
44150522001321431314
31405315442225344001 23450123450123450555
0413510022101233344 45052200132131344344
05345441225544001234
35100222012333344150
52200132143131431405 31544222554403011345
01134501234505550413 51001220123333441505
11001311431314314053
15442225544001234501 23450123450555041351
00112011333344150512
44121554400123450113







The Journey Ahead



As the Data Scientist wanders across the ill-defined boundary between Data Science and Machine Learning, in search of the fabled land of Artificial Intelligence, they find that the language changes from programming to a creole of linear algebra and probablity and statistics.

Deep Learning / Neural Nets

Without question the biggest thing in ML and computer science right now. Is the hype real? Can you learn anything meaningful in an afternoon? How did we get to this point?

The ideas have been around for decades. Two components came together in the past decade to enable astounding progress:

Widespread parallel computing (GPUs)

• Big data training sets





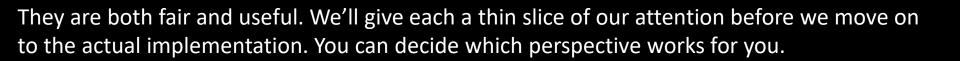
Two Perspectives

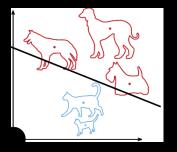
There are really two common ways to view the fundaments of deep learning.

• Inspired by biological models.



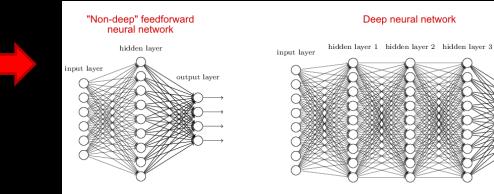
• An evolution of classic ML techniques (the perceptron).







Modeled After The Brain



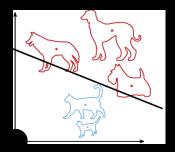


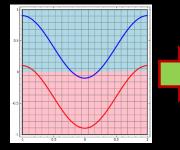
output layer

	0	1	0	0	0	0	1	0	0	1	0	0	0	1
	1	0	1	0	0	0	1	1	0	1	0	1	0	
	0	1	0	1	0	0	0	1	1	1	1	0	1	
	0	0	1	0	0	0	0	0	1	1	0	0	0	L
	0	0	0	0	0	1	0	0	0	0	0	1	0	
	0	0	1	1	0	0	0	0	0	1	0	0	0	L
M =	1	1	0	0	0	0	0	0	0	1	0	0	0	L
	0	1	1	0	0	0	0	0	0	1	1	1	1	
	0	0	0	0	1	0	0	0	0	0	0	1	0	L
	1	1	1	0	0	0	1	1	0	0	0	1	0	L
	0	0	1	0	0	0	0	1	0	0	0	0	1	
	0	1	1	0	1	0	0	1	1	1	0	0	0	
	0	0	1	0	0	0	0	1	0	0	1	0	0	L
	-													

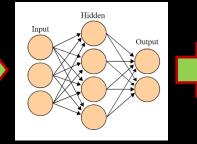
As a Highly Dimensional Non-linear Classifier

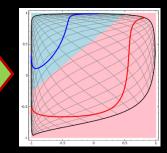
Perceptron





Network



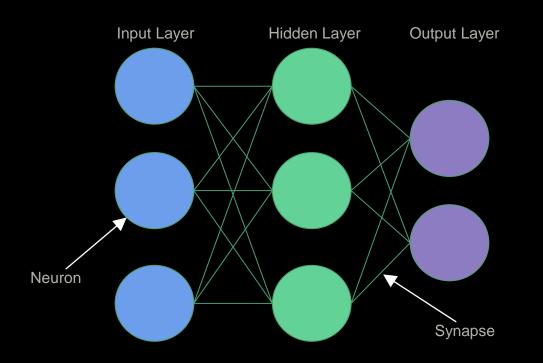


No Hidden Layer Linear

Hidden Layers Nonlinear

Courtesy: Chris Olah

Basic NN Architecture



In Practice

How many outputs?



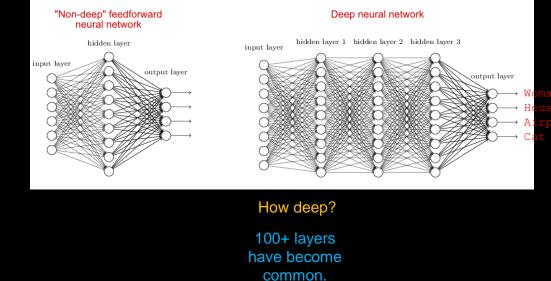
Might be an entire image.

Or could be discreet set of classification possibilities.

How many inputs?

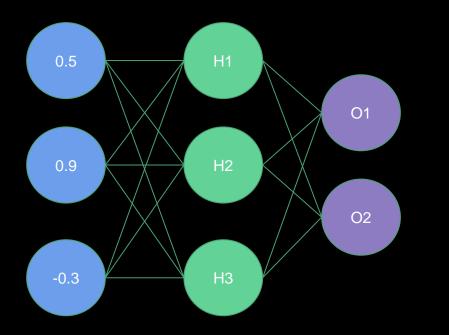


For an image it could be one (or 3) per pixel.



Inference

The "forward" or thinking step



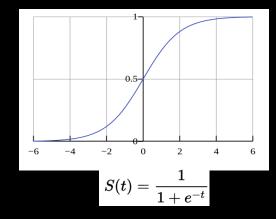
H1 Weights = (1.0, -2.0, 2.0) H2 Weights = (2.0, 1.0, -4.0) H3 Weights = (1.0, -1.0, 0.0)

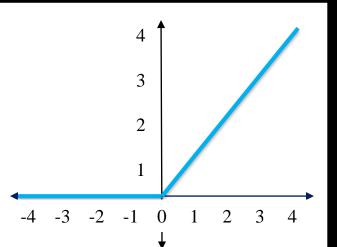
O1 Weights = (-3.0, 1.0, -3.0) O2 Weights = (0.0, 1.0, 2.0)

Activation Function

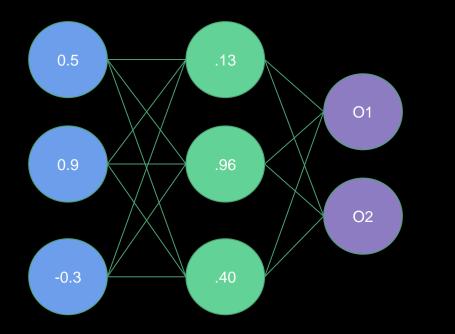
Neurons apply activation functions at these summed inputs. Activation functions are typically non-linear.

- The Sigmoid function produces a value between 0 and 1, so it is intuitive when a probability is desired, and was almost standard for many years.
- The Rectified Linear activation function is zero when the input is negative and is equal to the input when the input is positive. Rectified Linear activation functions are currently the most popular activation function as they are more efficient than the sigmoid or hyperbolic tangent.
 - Sparse activation: In a randomly initialized network, only 50% of hidden units are active.
 - Better gradient propagation: Fewer vanishing gradient problems compared to sigmoidal activation functions that saturate in both directions.
 - Efficient computation: Only comparison, addition and multiplication.
 - There are Leaky and Noisy variants.





Inference

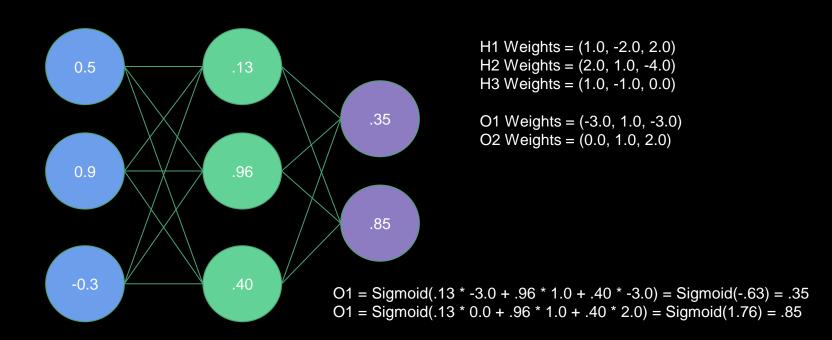


H1 Weights = (1.0, -2.0, 2.0) H2 Weights = (2.0, 1.0, -4.0) H3 Weights = (1.0, -1.0, 0.0)

O1 Weights = (-3.0, 1.0, -3.0) O2 Weights = (0.0, 1.0, 2.0)

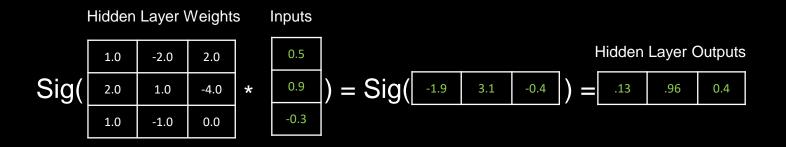
 $\begin{array}{l} \text{H1} = \text{Sigmoid}(0.5 * 1.0 + 0.9 * -2.0 + -0.3 * 2.0) = \text{Sigmoid}(-1.9) = .13 \\ \text{H2} = \text{Sigmoid}(0.5 * 2.0 + 0.9 * 1.0 + -0.3 * -4.0) = \text{Sigmoid}(3.1) = .96 \\ \text{H3} = \text{Sigmoid}(0.5 * 1.0 + 0.9 * -1.0 + -0.3 * 0.0) = \text{Sigmoid}(-0.4) = .40 \\ \end{array}$

Inference



As A Matrix Operation

H1 Weights = (1.0, -2.0, 2.0) H2 Weights = (2.0, 1.0, -4.0) H3 Weights = (1.0, -1.0, 0.0)

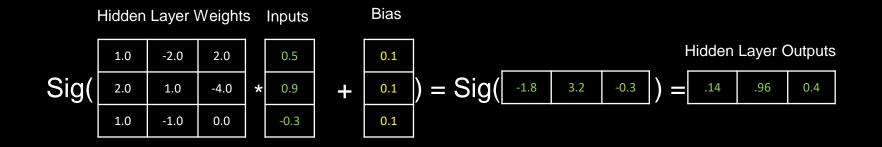


Now this looks like something that we can pump through a GPU.

Biases

It is also very useful to be able to offset our inputs by some constant. You can think of this as centering the activation function, or translating the solution (next slide). We will call this constant the *bias*, and it there will often be one value per layer.

Our math for the previously calculated layer now looks like this with **bias=0.1**:



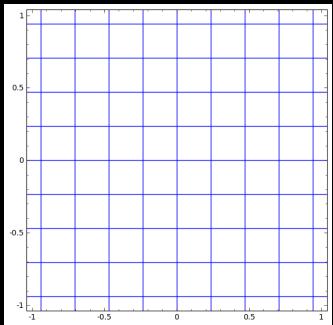
Linear + Nonlinear

The magic formula for a neural net is that, at each layer, we apply linear operations (which look naturally like linear algebra matrix operations) and then pipe the final result through some kind of final nonlinear activation function. The combination of the two allows us to do very general transforms.

The matrix multiply provides the *skew, rotation* and *scale*.

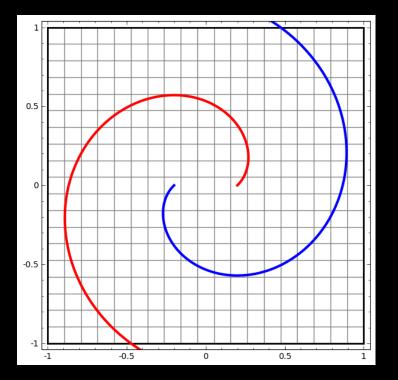
The bias provides the *translation*.

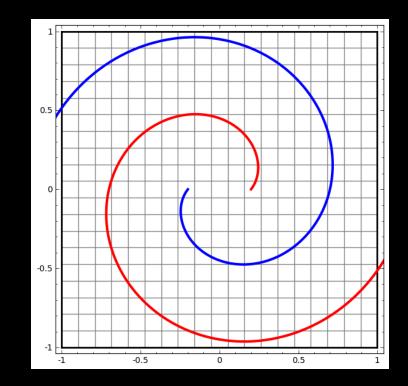
The activation function provides the *warp*.



Linear + Nonlinear

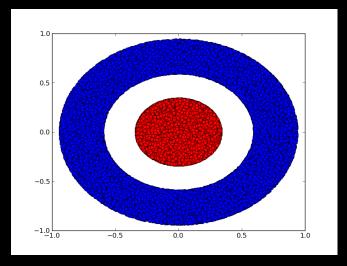
These are two very simple networks untangling spirals. Note that the second does not succeed. With more substantial networks these would both be trivial.





Width of Network

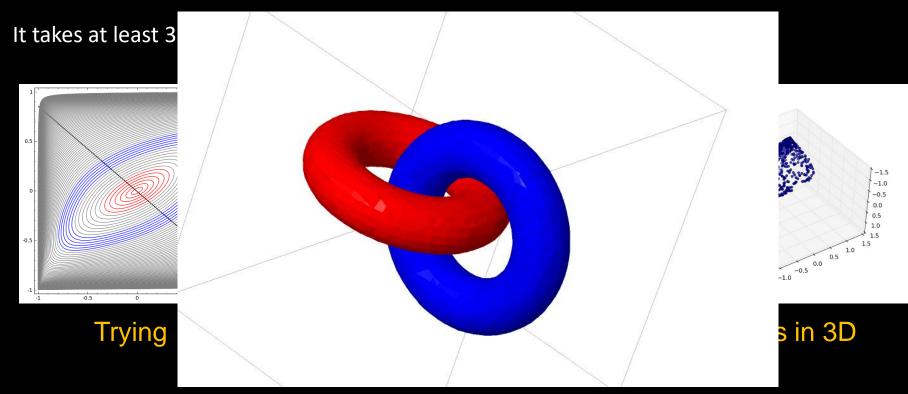
A very underappreciated fact about networks is that the width of any layer determines how many dimensions it can work in. This is valuable even for lower dimension problems. How about trying to classify (separate) this dataset:



Can a neural net do this with twisting and deforming? What good does it do to have more than two dimensions with a 2D dataset?

Courtesy: Chris Olah

Working In Higher Dimensions



Greater depth allows us to stack these operations, and can be very effective. The gains from depth are harder to characterize.

Courtesy: Chris Olah

Theoretically

Universal Approximation Theorem: A 1-hidden-layer feedforward network of this type can approximate any function¹, given enough width².

Not really that useful as:

- Width could be enormous.
- Doesn't tell us how to find the correct weights.

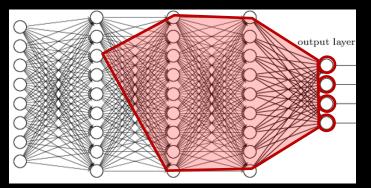
1) Borel measurable. Basically, mostly continuous and bounded.

2) Could be exponential number of hidden units, with one unit required for each distinguishable input configuration.

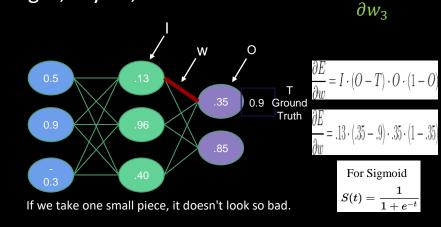
Training Neural Networks

So how do we find these magic weights? We want to minimize the error on our training data. Given labeled inputs, select weights that generate the smallest average error on the outputs.

We know that the output is a function of the weights: $E(w_1, w_2, w_3, ..., i_1, ..., t_1, ...)$. So to figure out which way, and how much, to push any particular weight, say w_3 , we want to calculate $\frac{\partial E}{\partial E}$



There are a lot of dependencies going on here. It isn't obvious that there is a viable way to do this in very large networks.



Note that the role of the gradient, $\frac{\partial E}{\partial w_3}$, here means that it becomes a problem if it vanishes. This is an issue for very deep networks.

Backpropagation

If we use the chain rule repeatedly across layers we can work our way backwards from the output error through the weights, adjusting them as we go. Note that this is where the requirement that activation functions must have nicely behaved derivatives comes from.

This technique makes the weight inter-dependencies much more tractable. An elegant perspective on this can be found from Chris Olah at http://colah.github.io/posts/2015-08-Backprop .

With basic calculus you can readily work through the details. You can find an excellent explanation from the renowned *3Blue1Brown* at

https://www.youtube.com/watch?v=Ilg3gGewQ5U .

You don't need to know the details, and this is all we have time to say, but you certainly can understand this fully if your freshman calculus isn't too rusty and you have some spare time.

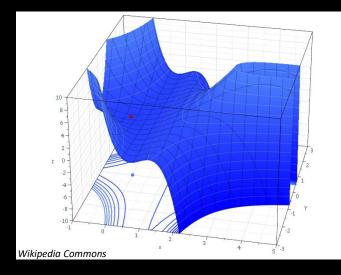
Solvers

However, even this efficient process leaves us with potentially many millions of simultaneous equations to solve (real nets have a lot of weights). They are non-linear to boot. Fortunately, this isn't a new problem created by deep learning, so we have options from the world of numerical methods.

The standard has been *gradient descent*. Methods, often similar, have arisen that perform better for deep learning applications. TensorFlow will allow us to use these interchangeably - and we will.

Most interesting recent methods incorporate *momentum* to help get over a local minimum. Momentum and *step size* are the two *hyperparameters* we will encounter later.

Nevertheless, we don't expect to ever find the actual global minimum.



We could/should find the error for all the training data before updating the weights (an *epoch*). However it is usually much more efficient to use a *stochastic* approach, sampling a random subset of the data, updating the weights, and then repeating with another *mini-batch*.

Going To Play Along?

Make sure you are on a GPU node:

bridges2-login014% interact -gpu
v001%

Load the TensorFlow 2 Container:

v001% singularity shell --nv /ocean/containers/ngc/tensorflow/tensorflow_21.02-tf2-py3.sif

And start TensorFlow:

```
Singularity> python
Python 3.8.5 (default, Jul 28 2020, 12:59:40)
[GCC 9.3.0] on linux
Type "help", "copyright", "credits" or "license"
>>> import tensorflow
>>> ...some congratulatory noise...
>>>
```

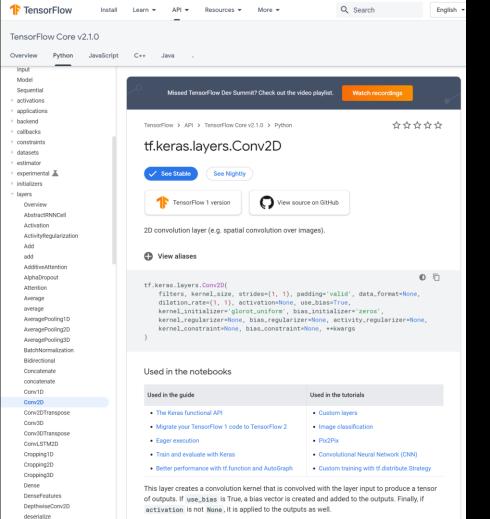
Two Other Ways To Play Along

From inside the container, and in the right example directory, run the python programs from the command line:

Singularity> python CNN_Dropout.py

or invoke them from within the python shell:

>>> exec(open("./CNN_Dropout.py").read())



When using this layer as the first layer in a model, provide the keyword argument input_shape (tuple of integers, does not include the sample axis), e.g. input_shape=(128, 128, 3) for

Dot

Documentation

The API is well documented.

That is terribly unusual.

Take advantage and keep a browser open as you develop.

MNIST

We now know enough to attempt a problem. Only because the TensorFlow framework, and the Keras API, fills in a lot of the details that we have glossed over. That is one of its functions.

Our problem will be character recognition. We will learn to read handwritten digits by training on a large set of 28x28 greyscale samples.



First we'll do this with the simplest possible model just to show how the TensorFlow framework functions. Then we will gradually implement our way to a quite sophisticated and accurate convolutional neural network for this same problem.

Getting Into MNIST

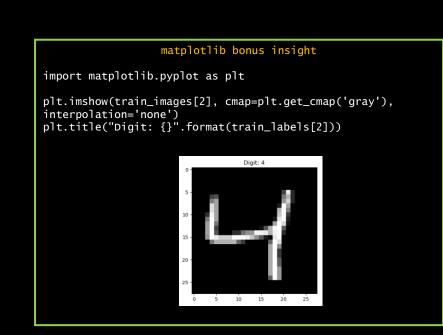
import tensorflow as tf

mnist = tf.keras.datasets.mnist
(train_images, train_labels), (test_images, test_labels) = mnist.load_data()

train_images = train_images.reshape(60000, 784)
test_images = test_images.reshape(10000, 784)

test_images = test_images.astype('float32')
train_images = train_images.astype('float32')

test_images /= 255
train_images /= 255



Defining Our Network

```
import tensorflow as tf
mnist = tf.keras.datasets.mnist
(train_images, train_labels), (test_images, test_labels) = mnist.load_data()
train_images = train_images.reshape(60000, 784)
test_images = test_images.reshape(10000, 784)
test images = test images.astvpe('float32')
train_images = train_images.astype('float32')
test_images /= 255
train_images /= 255
model = tf.keras.Sequential([
  tf.keras.layers.Dense(64, activation='relu', input_shape=(784,)),
  tf.keras.layers.Dense(64, activation='relu'),
  tf.keras.layers.Dense(10, activation='softmax'),
])
model.compile(optimizer='adam', loss='sparse_categorical_crossentropy', metrics=['accu
```

Starting from zero?

In general, initialization values are hard to pin down analytically. Values might help optimization but hurt generalization, or vice versa.

The only certainty is you need to have different values to break the symmetry, or else units in the same layer, with the same inputs, would track each other.

Practically, we just pick some "reasonable" values.

	<pre>model.summary()</pre>					
rac	Layer (type)	Output	Shape	Param #		
	dense_6 (Dense)	(None,	64)	50240		
	dense_7 (Dense)	(None,	64)	4160		
	dense_8 (Dense)	(None,	10)	650		
	Total params: 55,050 Trainable params: 55,050 Non-trainable params: 0					

Cross Entropy Loss & Softmax

Why Softmax?								
The values coming out of our matrix operations can have large, and negative values. We would like our solution vector to be conventional probabilities that sum to 1.0. An effective way to normalize our outputs is to use the popular <i>Softmax</i> function. Let's look at an example with just three possible digits:								
Digit	Output	Exponential	Normalized					
0 1 2	4.8 -2.6 2.9	121 0.07 18	.87 .00 .13					

Given the sensible way we have constructed these outputs, the Cross Entropy Loss function is a good way to define the error across all possibilities. Better than squared error, which we have been using until now. It is defined as $-\Sigma y_{-} \log y$, or if this really is a 0, $y_{-}=(1,0,0)$, and

 $-1\log(0.87) - 0\log(0.0001) - 0\log(0.13) = -\log(0.87) = -0.13$

You can think that it "undoes" the Softmax, if you want.

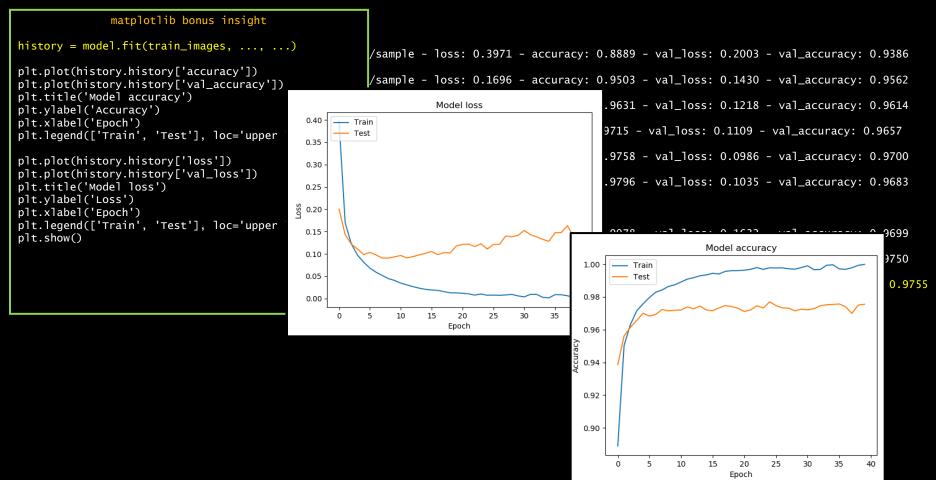
Training

```
import tensorflow as tf
mnist = tf.keras.datasets.mnist
(train_images, train_labels), (test_images, test_labels) = mnist.load_data()
train_images = train_images.reshape(60000, 784)
test_images = test_images.reshape(10000, 784)
test_images = test_images.astype('float32')
train_images = train_images.astype('float32')
test_images /= 255
train_images /= 255
model = tf.keras.Sequential([
  tf.keras.layers.Dense(64, activation='relu', input_shape=(784,)),
  tf.keras.layers.Dense(64, activation='relu'),
  tf.keras.layers.Dense(10, activation='softmax'),
])
```

model.compile(optimizer='adam', loss='sparse_categorical_crossentropy', metrics=['accuracy'])

history = model.fit(train_images, train_labels, batch_size=128, epochs=40, verbose=1, validation_data=(test_images, test_labels))

Results



Let's Go Wider

```
import tensorflow as tf

mnist = tf.keras.datasets.mnist
(train_images, train_labels), (test_images, test_labels) = mnist.load_data()

train_images = train_images.reshape(60000, 784)
test_images = test_images.reshape(10000, 784)

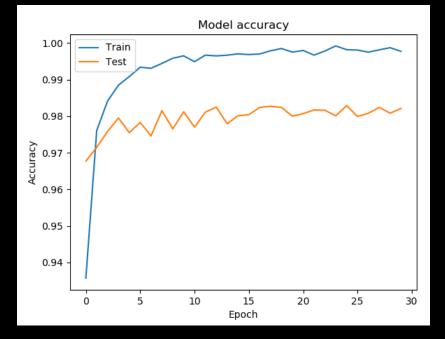
test_images = test_images.astype('float32')
train_images = train_images.astype('float32')
test_images /= 255
train_images /= 255
model = tf.keras.sequential([
    tf.keras.layers.Dense(512, activation='relu', input_shape=(784,)),
    tf.keras.layers.Dense(10, activation='softmax'),
])
```

model.compile(optimizer='adam', loss='sparse_categorical_crossentropy', metrics=['accuracy'])

model.fit(train_images, train_labels, batch_size=128, epochs=30, verbose=1, validation_data=(test_images, test_labels))

Wider Results

Epoch 30/30



Wider										
model.summary()										
Layer (type))	Output	Shape	Param #						
dense_18 (De	ense)	(None,	512)	401920						
dense_19 (D	ense)	(None,	512)	262656						
dense_20 (D	ense)	(None,	10)	5130						
Total params: 669,706 Trainable params: 669,706 Non-trainable params: 0										
55,050 for 64 wide Model										

Maybe Deeper?

```
import tensorflow as tf
mnist = tf.keras.datasets.mnist
(train_images, train_labels), (test_images, test_labels) = mnist.load_data()
train_images = train_images.reshape(60000, 784)
test_images = test_images.reshape(10000, 784)
test_images = test_images.astype('float32')
train_images = train_images.astype('float32')
test_images /= 255
train_images /= 255
model = tf.keras.sequential([
    tf.keras.layers.Dense(512, activation='relu', input_shape=(784,)),
    tf.keras.layers.Dense(512, activation='relu'),
    tf.keras.layers.Dense(10, activation='relu'),
    tf.keras.layers.Dense(10, activation='softmax'),
])
```

model.compile(optimizer='adam', loss='sparse_categorical_crossentropy', metrics=['accuracy'])

model.fit(train_images, train_labels, batch_size=128, epochs=30, verbose=1, validation_data=(test_images, test_labels))

Wide And Deep Results

. . . .

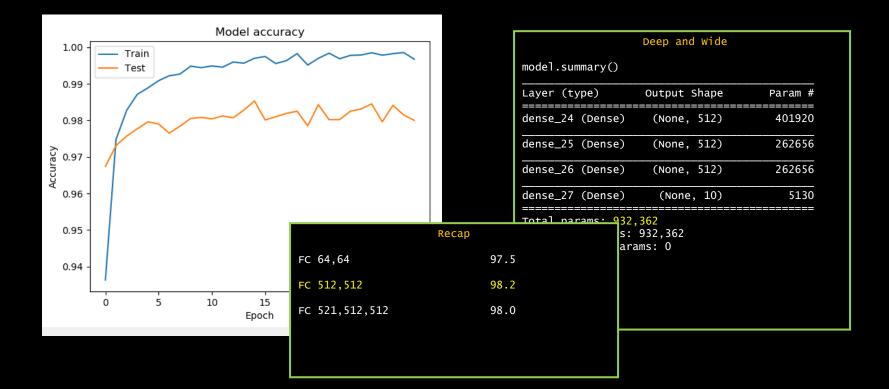


Image Recognition Done Right: CNNs

AlexNet won the 2012 ImageNet LSVRC and changed the DL world.

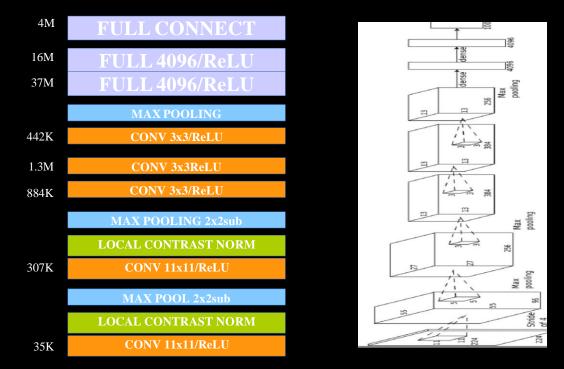
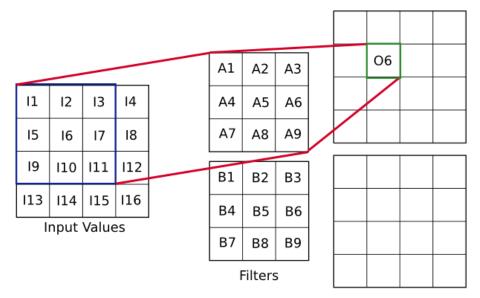


Image Object Recognition [Krizhevsky, Sutskever, Hinton 2012]

Convolution

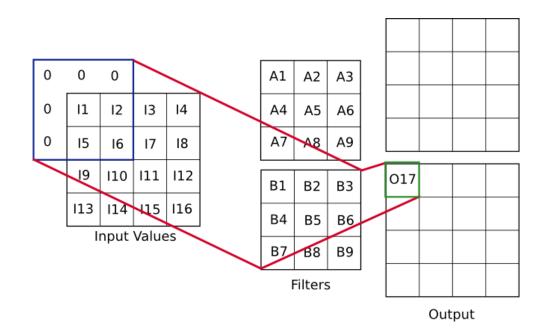


Output

 $\begin{array}{l} O_6 = A_1 \cdot I_1 + A_2 \cdot I_2 + A_3 \cdot I_3 \\ + A_4 \cdot I_5 + A_5 \cdot I_6 + A_6 \cdot I_7 \\ + A_7 \cdot I_9 + A_8 \cdot I_{10} + A_9 \cdot I_{11} \end{array}$

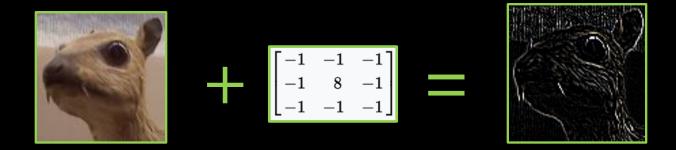
Convolution

Boundary and Index Accounting



 $O_{17} = B_5 \cdot I_1 + B_6 \cdot I_2 + B_8 \cdot I_5 + B_9 \cdot I_6$

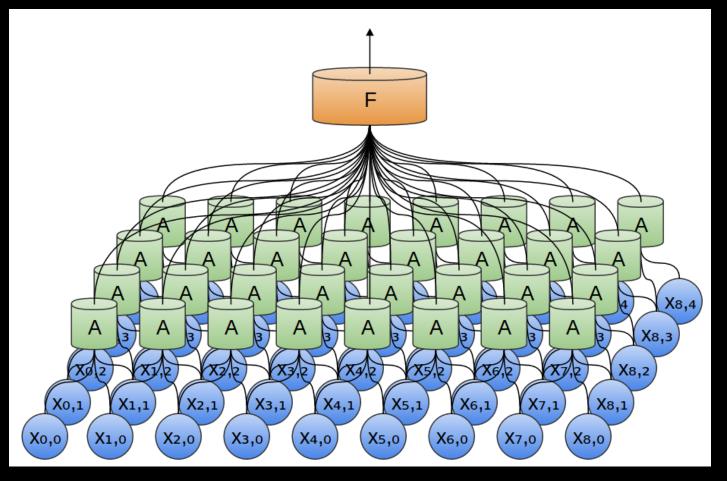
Straight Convolution



Edge Detector

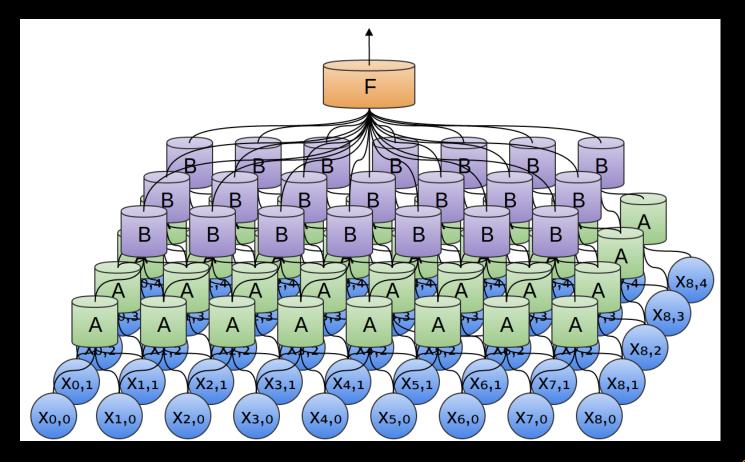
Images: Wikipedia

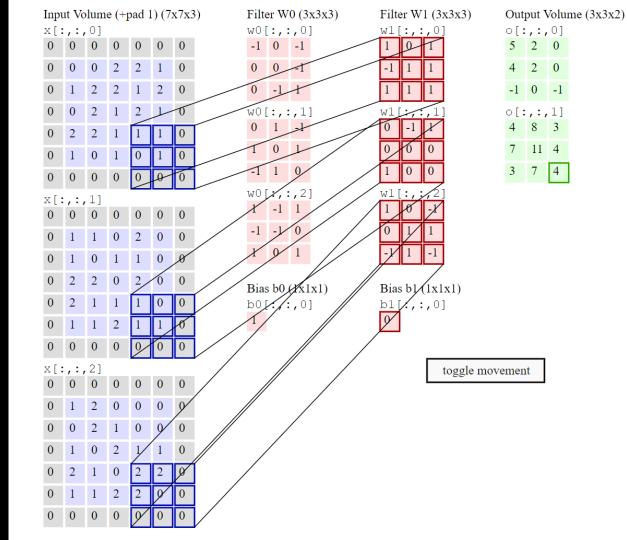
Simplest Convolution Net



Courtesy: Chris Olah

Stacking Convolutions





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From the very nice Stanford CS231n course at <u>http://cs231n.gith</u> <u>ub.io/convolution</u> <u>al-networks/</u>

Stride = 2

Convolution Math

Each Convolutional Layer:

Inputs a volume of size $W_1 \times H_1 \times D_1$ (D is depth)

Requires four hyperparameters:

Number of filters K their spatial extent N the stride S the amount of padding P

```
Produces a volume of size W_0 \times H_0 \times D_0

W_0 = (W_1 - N + 2P) / S + 1

H_0 = (H_1 - F + 2P) / S + 1

D_0 = K
```

This requires $N \cdot N \cdot D_1$ weights per filter, for a total of $N \cdot N \cdot D_1 \cdot K$ weights and K biases

In the output volume, the d-th depth slice (of size $W_0 \times H_0$) is the result of performing a convolution of the d-th filter over the input volume with a stride of S, and then offset by d-th bias.